Week 7 Worksheet Thursday

Instructions. Discuss with your group mates and do the following problems. You are not expected to finish all the problems. :)

Topic: Derivative Computing

Enhance the impression! Common Derivatives that we learned so far:

$(x^n)' =$ nx^{n-1} $\qquad \frac{d}{dx}[\sin x] =$ $\cos x$

$\frac{d\cos x}{dx} =$ $-\sin x$ $\qquad (\tan x)' =$ $\sec^2 x = \frac{1}{\cos^2 x}$

1. Compute the Derivatives of the following functions (No need to use definition.)

   (a) $f(x) = \frac{1}{\sqrt{1+\sqrt{x}}}$

   (b) $y = (x+x^2)^2 (\tan x)^3$

   (c) $y = \frac{\cos(2x)}{\pi + x^2}$

   (d) $y = \sin(3x) - \cos(2x)$

   (e) $f(x) = (\cos x)^3 - \cos(x^3)$

   (f) $y = \cos(\sin(\cos(x^2)))$

   (g) $f(x) = \cos(x^2)\tan(\sqrt{x+1})$ *(2013fall exam)*

   (h) Consider y as a function of x. What is $\frac{d[y^2]}{dx}$ and $\frac{d}{dx}[xy]$? (The expression should contain $\frac{dy}{dx}$)

(a) $f(x) = (1+x^{\frac{1}{2}})^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{2}(1+x^{\frac{1}{2}})^{-\frac{3}{2}} \frac{1}{2}x^{-\frac{1}{2}}$

(b) $y' = 2(x+x^2)(1+2x)(\tan x)^3 + (x+x^2)^2\, 3(\tan x)^2 \sec^2 x$

(c) $y = \frac{\cos(2x)}{\pi + x^2}$

$y' = \frac{-2\sin(2x)\cdot(\pi+x^2) - 2x\cos(2x)}{(\pi+x^2)^2}$

(d) $y' = 3\cos(3x) + 2\sin(2x)$

(e) $f'(x) = 3(\cos x)^2(-\sin x) + \sin(x^3)\cdot 3x^2$

(f) $y' = -\sin(\sin(\cos(x^2)))\cdot\cos(\cos(x^2))\cdot(-\sin(x^2))\cdot 2x$

(g) $f'(x) = -\sin(x^2)\cdot 2x\tan(\sqrt{x+1}) + \cos(x^2)\sec^2(\sqrt{x+1})\,\frac{1}{2}(x+1)^{-\frac{1}{2}}$

(h) $\frac{d[y^2]}{dx} = 2y\frac{dy}{dx}$ $\qquad \frac{d[xy]}{dx} = y + x\frac{dy}{dx}$

2. $f(x) = 3\sin(-2x) + 500x^{2015} + x^{2016}$. What is $f^{(2016)}(x)$?

① 2016th derivative of $500x^{2015}$ will be 0.

② Consider $g(x) = 3\sin(-2x)$

$g'(x) = 3 \cos(-2x) \cdot (-2)$

$g''(x) = 3\left[-\sin(-2x)\right](-2)^2$

$g^{(3)}(x) = 3\left[-\cos(-2x)\right](-2)^3$

$g^{(4)}(x) = 3\left[\sin(-2x)\right](-2)^4$

$\cdots$

$g^{(2016)}(x) = 3\sin(-2x)(-2)^{2016}$

③ Consider $h(x) = x^{2016}$

④ $h'(x) = 2016x^{2015}$

$h''(x) = 2016 \cdot 2015 \cdot x^{2014}$

$h^{(3)}(x) = 2016 \cdot 2015 \cdot 2014\, x^{2013}$

$\cdots$

$h^{(2016)}(x) = 2016 \cdot 2015 \cdot 2014 \cdot \cdots \cdot 1$

$= 2016!$

So $f^{(2016)}(x) = 3\sin(-2x)(-2)^{2016} + 2016!$